1) $A = \begin{bmatrix} -4 & -6 & 3 \\ 2 & 4 & -2 \\ -2 & -2 & 1 \end{bmatrix}$

a) Busco los λ tq $\det(\lambda I - A) = 0$

$$\to \det\left(\begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} -4 & -6 & 3 \\ 2 & 4 & -2 \\ -2 & -2 & 1 \end{bmatrix}\right) = 0$$

$$\to \det\left(\begin{bmatrix} \lambda+4 & 6 & -3 \\ -2 & \lambda-4 & 2 \\ 2 & 2 & \lambda-1 \end{bmatrix}\right) = 0$$

$$\to (\lambda+4) \cdot \begin{vmatrix} \lambda-4 & 2 \\ 2 & \lambda-1 \end{vmatrix} + 2 \cdot \begin{vmatrix} 6 & -3 \\ 2 & \lambda-1 \end{vmatrix} + 2 \cdot \begin{vmatrix} 6 & -3 \\ \lambda-4 & 2 \end{vmatrix} = 0$$

$$\to (\lambda+4) \cdot \left[(\lambda^2 - 5\lambda + \cancel{4}) - \cancel{4}\right] + 2 \cdot (6\lambda - \cancel{6} + \cancel{6}) + 2 \cdot \overbrace{\left(\cancel{12} - (-3\lambda + \cancel{12})\right)}^{3\lambda} = 0$$

$$\to (\lambda+4) \cdot (\lambda^2 - 5\lambda) + 12\lambda + 6\lambda = 0$$

$$\to \lambda^3 - 5\lambda^2 + 4\lambda^2 - 20\lambda + 12\lambda + 6\lambda = 0$$

$$\to \lambda^3 - \lambda^2 - 2\lambda = 0 \to \lambda \cdot \underbrace{(\lambda^2 - \lambda - 2)}_{\frac{1 \pm \sqrt{1+8}}{2} \nearrow 2, \searrow -1} = 0$$

$$\boxed{\lambda_1 = 0, \quad \lambda_2 = 2, \quad \lambda_3 = -1}$$

Para los autovectores de cada λ, busco los $v \neq 0$ tales que: ~~$(\lambda I - A)v$~~

$(\lambda I - A) . v = 0$

Para $\lambda = 0$:

$$\begin{bmatrix} 4 & 6 & -3 \\ -2 & -4 & 2 \\ 2 & 2 & -1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \bar{0}$$

$$\begin{pmatrix} 4 & 6 & -3 \\ -2 & -4 & 2 \\ 2 & 2 & -1 \end{pmatrix} \begin{matrix} F2 \to F1+2F2 \\ F3 \to F1-2F3 \end{matrix} \begin{pmatrix} 4 & 6 & -3 \\ 0 & -2 & 1 \\ 0 & 2 & -1 \end{pmatrix} F3 \to F2+F3 \begin{pmatrix} 4 & 6 & -3 \\ 0 & -2 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\to \begin{cases} 4x + 6y - 3z = 0 \to 4x + \cancel{6y} - \cancel{6y} = 0 \to x = 0 \\ -2y + z = 0 \to z = 2y \end{cases}$$

$\to \bar{X} = (0; y; 2y) = y . (0; 1; 2)$

$\boxed{\text{Autovector: } (0;1;2) \quad \lambda = 0}$

Para $\lambda = -1$

$$\begin{bmatrix} 3 & 6 & -3 \\ -2 & -5 & 2 \\ 2 & 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \bar{0}$$

$$\begin{pmatrix} 3 & 6 & -3 \\ -2 & -5 & 2 \\ 2 & 2 & -2 \end{pmatrix} \begin{matrix} F2 \to 2F1+3F2 \\ F3 \to 2F1-3F3 \end{matrix} \begin{pmatrix} 3 & 6 & -3 \\ 0 & -3 & 0 \\ 0 & 6 & 0 \end{pmatrix} F3 \to 2F2+F3 \begin{pmatrix} 3 & 6 & -3 \\ 0 & -3 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\to \begin{cases} 3x + 6y - 3z = 0 \to x = z \\ -3y = 0 \to y = 0 \end{cases} \to \bar{X} = (z; 0; z) = z . (1; 0; 1)$$

$\boxed{\text{AUTOVECTOR } \lambda = -1 : (1;0;1)}$

Para $\lambda = 2$

$$\begin{pmatrix} 6 & 6 & -3 \\ -2 & -2 & 2 \\ 2 & 2 & 1 \end{pmatrix} \begin{matrix} F2 \to F1 + 3F2 \\ F3 \to F1 - 3F2 \end{matrix} \begin{pmatrix} 6 & 6 & -3 \\ 0 & 0 & 3 \\ 0 & 0 & -6 \end{pmatrix} F3 \to 2F2 + F3 \begin{pmatrix} 6 & 6 & -3 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\to \begin{cases} 6x + 6y - 3z = 0 \to x = -y \\ 3z = 0 \to z = 0 \end{cases} \to \bar{X} = (-y; y; 0) = y \cdot (-1; 1; 0)$$

$\boxed{\text{AUTOVECTOR } \lambda = 2 : (-1; 1; 0)}$

b) Verifico que son base:

$$\begin{array}{c} v1 \\ v2 \\ v3 \end{array} \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix} \xrightarrow{v1 \leftrightarrow v2} \begin{array}{c} v2 \\ v1 \\ v3 \end{array} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ -1 & 1 & 0 \end{pmatrix} F3 \to F1 + F3 \begin{array}{c} v2 \\ v1 \\ v3 \end{array} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 1 \end{pmatrix} F3 \to F2 - F3$$

$$\begin{array}{c} v2 \\ v1 \\ v3 \end{array} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

claramente son LI ya que triangulé y no se anuló ninguna. → son base ✓

c) $B = \{v1, v2, v3\} = \{(0,1,2), (1,0,1), (-1,1,0)\}$.

$[v1]^E = (0,1,2)$ $\quad [v2]^E = (1,0,1)$ $\quad [v3]^E = (-1,1,0)$

$$\to \boxed{M_B^E = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}}$$